1. Which of the following is equivalent to $\left(\dfrac{(x^5)^{-2}}{\left(\frac{1}{x^4}\right)^3}\right)^{-5}$, assuming $x \neq 0$?

    (A) $\dfrac{1}{x^{10}}$ (B) x^{10} (C) x^{110} (D) x^{38} (E) x^{62}

2. What is the smallest integer m for which $\dfrac{10!}{m}$ is a perfect square?

    (A) 2 (B) 10 (C) 7 (D) 28 (E) 35

3. Half the books on a teacher's bookshelf are mathematics books, a fifth of them are physics books, and a tenth of them are history books. The remainder of the books are romance novels. If 4 of the mathematics books and 5 of the physics books are replaced by romance novels, then romance novels will comprise 30% of the books on the bookshelf. How many books, in total, are on the bookshelf?

    (A) 60 (B) 120 (C) 80 (D) 160 (E) 90

4. Samwise determines that since his final exam counts as two tests, he only needs to score a 28 on it for his test average to be 70. If he gets a perfect 100 on the final exam, his average will be 88. What is the lowest score Samwise can receive on his final and still have an average of 80?

    (A) 65 (B) 66 (C) 68 (D) 70 (E) 72

5. How many divisors of 630 are multiples of 3?

    (A) 8 (B) 10 (C) 12 (D) 16 (E) 24

6. At noon, Milly starts walking from her house to the park at a rate of 2 miles per hour. Fifteen minutes later, Milly's brother Jordan starts jogging from the house to the park at a rate of 6 miles per hour. If Jordan and Milly take the same path, how many minutes will it take Jordan to catch up with Milly? (Assume that Jordan catches up with Milly before they reach the park).

    (A) 7.5 (B) 8 (C) 10 (D) 12.5 (E) 15

7. Triangle AHI is equilateral. Segments $\overline{BC}$, $\overline{DE}$ and $\overline{FG}$ are all parallel to $\overline{HI}$ and $AB = BD = DF = FH$. What is the ratio of the area of trapezoid $FGIH$ to the area of triangle AHI?
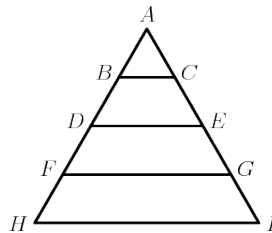

    (A) $\dfrac{1}{4}$ (B) $\dfrac{7\sqrt{3}}{16}$ (C) $\dfrac{7}{16}$ (D) $\dfrac{\sqrt{3}}{3}$ (E) $\dfrac{9}{25}$

8. Jackson has a deck of cards, where each card is labeled with some integer from 1 to 20, inclusive. The number on the card indicates how many cards there are in the deck with that number. Every integer from 1 to 20 (inclusive) appears in the deck, except for one number. If the probability of selecting a 12 at random from the deck is 6%, what number is missing from the deck?

    (A) 6 (B) 7 (C) 9 (D) 10 (E) 15

9. What is the sum of the series below?

$$\frac{1}{4} - \frac{1}{5} + \frac{1}{16} - \frac{1}{25} + \frac{1}{64} - \frac{1}{125} + \cdots$$

   (A) $\dfrac{1}{9}$ (B) $\dfrac{1}{12}$ (C) $\dfrac{1}{7}$ (D) $\dfrac{1}{20}$ (E) $\dfrac{7}{12}$

10. Right triangle ABC has legs of length 5 and 12. A semicircle is inscribed in the triangle, as shown below. What is the radius of the semicircle?
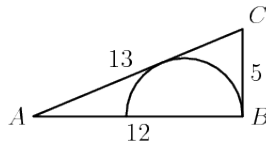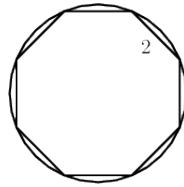

   (A) $2\sqrt{3}$ (B) $2\sqrt{2}$ (C) $\dfrac{5}{2}$ (D) $\dfrac{13}{5}$ (E) $\dfrac{10}{3}$

11. The first three terms in an arithmetic sequence have a sum of 60. The next three terms in the same sequence have a sum of 105. What is the 2014th term in the sequence?

   (A) 10,064   (B) 10,080   (C) 10,078   (D) 10,084   (E) 10,075

12. A regular octagon with side length 2 is inscribed in a circle, as shown below. What is the area of the circle?

   (A) 8π (B) 10π (C) $(8+4\sqrt{2})\pi$ (D) $(5+2\sqrt{2})\pi$ (E) $(4+2\sqrt{2})\pi$

13. The operation $\#$ is defined by $x\#y = \dfrac{x-y}{xy}$. For how many real values a is $a\#(a\#2) = 1$?

   (A) 0   (B) 1   (C) 2   (D) 4   (E) infinitely many

14. A group of 6 friends sit in the back row of an otherwise empty movie theater. Each row in the theater contains 8 seats. Abby and Bellina are best friends, so they must sit next to each other, with no empty seat between them. However, Carlita called them names at lunch, so she cannot sit in an adjacent seat to either Abby or Bellina. In how many different ways can the 6 friends be seated in the back row?

   (A) 2520   (B) 3600   (C) 5040   (D) 5760   (E) 7200

15. The sum

$$\frac{1}{1+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}$$

   is a root of the quadratic $x^2 + x + c$. What is c in terms of n?

   (A) $-\frac{n}{2}$ (B) $2n$ (C) $-2n$ (D) $n+\frac{1}{2}$ (E) $n-2$

1. A
2. C
3. E
4. C
5. D
6. A
7. C
8. D
9. B
10. E
11. B
12. E
13. A
14. B
15. A

1. Find the remainder when 2^{2014} is divided by 7.

2. In base 10, 31×33 does not equal 1243. In what base does $31 \times 33 = 1243$?

3. Lines from the vertices of a unit square are drawn to the midpoints of the sides as shown in the figure below. What is the area of quadrilateral $ABCD$? Express your answer in simplest terms.
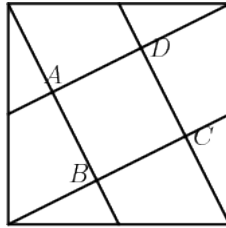
4. The 48 faces of 8 unit cubes are painted white. What is the smallest number of these faces that can be repainted black so that it becomes impossible to arrange the 8 unit cubes into a two by two by two cube, each of whose 6 faces is totally white?

5. Marcy rolls a standard six-sided die three times, and writes down the result of each roll to create a three-digit number. For example, if Marcy rolls a 1, then a 4, then a 5, her number will be 145. What is the probability that Marcy's number is divisible by 9? Express your answer as a fraction in simplest terms.

6. If $|x| - x + y = 10$ and $x + |y| + y = 12$, then what is the value of $x + y$? Express your answer in simplest terms.

7. A semicircle is joined to the side of a triangle, with the common edge removed. Sixteen points are arranged on the figure, as shown below. How many non-degenerate triangles can be drawn from the given points?

8. A right rectangular prism is inscribed within a sphere. The total area of all the faces the prism is 88, and the total length of all its edges is 48. What is the surface area of the sphere? Express your answer in terms of π.

1. 2
2. 8
3. $\frac{1}{5}$ or 0.2
4. 2
5. $\frac{13}{108}$
6. $\frac{26}{5}$ or 5.2
7. 540
8. 56π